### 1. Error in Pb fractionation factor (*FPb25*) for double spiked (202Pb-205Pb) samples

Because of the artificial nature of the 202Pb and 202Pb tracer isotopes, a unique algebraic expression for the linear lead fractionation factor as the coefficient per unit mass difference can be derived from the relationship:

  (1)

where *RPb25m* is the measured 202Pb/202Pb, and the tracer 202Pb/202Pb, *RPb25t,* is by definition equal to the mass fractionation corrected ratio. Solving for *FPb25* yields:

  (2)

The error propagation equation for *FPb25* may be written as (assuming all errors are uncorrelated):

  (3)

The partial derivatives are calculated as:

  (4)

  (5)

These partial derivatives and the appropriate variances can then be substituted into eqn. (3) to derive the uncertainty in the lead fractionation factor for double spiked (202Pb-205Pb) samples.

### 2. Error in U fractionation factor (*FU36*) for double spiked (233U-236U) samples

Because of the artificial nature of the 233U and 236U tracer isotopes, a simple algebraic expression for the linear uranium fractionation factor, *FU36*, as the coefficient per unit mass difference can be derived from the relationship:

  (6)

where *RU36m* is the measured 233U/236U, and the tracer 233U/236U, *RU36t,* is by definition equal to the mass fractionation corrected ratio. Solving for *FU36* yields:

  (7)

The error propagation equation for *FU36* may be written as (assuming all errors are uncorrelated):

  (8)

The partial derivatives are calculated as:

  (9)

  (10)

These partial derivatives and the appropriate variances can then be substituted into eqn. (8) to derive the uncertainty in the lead fractionation factor for double spiked (233U-236U) samples.

### 3. 238U sample from isotope dilution against 236U

First establishing the algebraic expression for sample 238U,

  (11)

the error propagation equation may be written as:

  (12)

Note that to dramatically simplify our derivation, we are considering error correlations between the component terms of sample 238U (*U238s*) and the uranium fractionation factor (*FU*) to be trivial, which is justified given the contrasting dominant error sources in each quantity. The partial derivatives are calculated as follows:

  (13)

  (14)

  (15)

  (16)

These partial derivatives and variances can then be substituted into eqn. 12 to derive the uncertainty in sample 238U.

### 4. 238U sample from isotope dilution against 233U

First establishing the algebraic expression for sample 238U,

  (17)

the error propagation equation may be written as:

  (18)

Note that to dramatically simplify our derivation, we are considering error correlations between the component terms of sample 238U (*U238s*) and the uranium fractionation factor (*FU*) to be trivial, which is justified given the contrasting dominant error sources in each quantity. The partial derivatives are calculated as follows:

  (19)

  (20)

  (21)

  (22)

These partial derivatives and variances can then be substituted into eqn. 18 to derive the uncertainty in sample 238U.